Telescope Design and Efficiency

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ABSTRACT

The cost effectiveness of modern telescope operations depends upon appropriate telescope system design. We explore telescope operating models and define the efficiency of telescope usage, using recent operational data. We investigate the efficiency of telescope use with respect to several generic types of operational programme. We derive a model of telescope operating efficiency and explore the operational implications of several telescope design factors and configurations.

Keywords: Telescope design, efficiency, availability, signal-to-noise

1. INTRODUCTION

When considering the design of professional astronomical telescopes it is important to offer sufficient and cost-effective technical solutions based upon a proper understanding of the scientific programme the telescope is intended to perform. To be able to offer such technical solutions a quantitative analysis of the design implications of the scientific programme must be performed. To optimise telescope design parameters requires a "fitting function" of some form to be explored and optimised numerically. We propose a definition of telescope efficiency that can be used for design optimisation and explore some implications of this definition.

2. OPERATING MODELS FOR CURRENT TELESCOPES

We have analysed operational data from log files created on the Isaac Newton 2.5m Telescope (INT) and the William Herschel 4.2m Telescope (WHT) at the Isaac Newton Group of Telescopes, La Palma (ING). The period during which data was collected was from 1997 August 1 to 1999 July 31 (corresponding to ING observing Semesters 97b, 98a, 98b, 99a). The log data include type of observation (including instrument, science or calibration data), object position, UT of the start of the observation, air mass at the start of the observation, exposure time, and some key instrument set-up data. Also, each log file presents a brief summary of weather conditions for the night and any technical down time experienced.

ING allocate telescope time to successful science proposals on the basis of season and phase of moon. A three phase lunar model is used to schedule observing at the ING: *Dark*, *Grey* and *Bright*, which roughly correspond to the following phases of the moon.

$$Dark : phase \leq 0.35 \tag{1}$$

$$Grey : 0.35 < phase < 0.65 \tag{2}$$

$$Bright : phase \geq 0.65 \tag{3}$$

We have analysed these log data to answer the question: how does the phase of the moon affect the kind of programme the telescope will perform? In particular, we are interested in the following:

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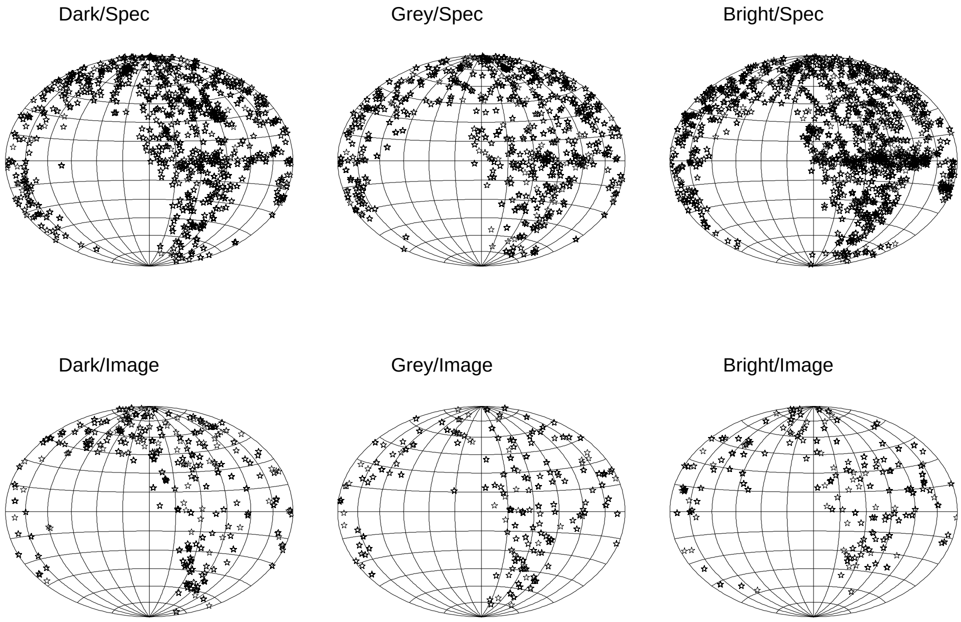

Figure 1. A plot of all WHT science observations during the period 1997 August 1 to 1999 July 31, presented in Galactic coordinates. The projection is Aitoff and has the Galactic centre at the centre of each plot. The plots classify the data into *Dark*, *Grey* and *Bright* phase of the moon and into imaging and spectroscopy.

1. How does the distribution of observed objects in the sky vary with the phase of the moon?
2. What instrumentation is used during nights of each lunar phase?
3. What is the mean exposure time during each lunar phase?

2.1. Distribution of observed objects

Figure 1 presents plots of all science observations made with the WHT during the period 1997 August 1 to 1999 July 31, categorised by phase of moon and by type of observation. Here we define a science observation as being any on-sky observation between the hours of Nautical twilight, including targets such as photometric and spectrophotometric standards, but excluding instrument calibration data such as flat field images, bias frames and arc spectra. We distinguish only between two types of observation: imaging and spectroscopy. These two broad categories represent the way in which the detector is likely to be being used to record the data; *i.e.* often sky background limited for imaging and readout noise limited for spectroscopy.

In **Figure 1** the large area where no data are plotted represents the area of sky not visible from La Palma ($17°52'.9W$, $+28°45'.6$). From the data plotted in **Figure 1** it can be seen that the distribution of observations about the sky is approximately homogeneous for imaging. For spectroscopy, two distributions are apparent – an approximately homogeneous distribution similar to that of the imaging observations and an additional distribution concentrated about the Galactic plane. The Galactic plane observations are clearly more abundant during *Bright* time.

Figure 2 presents plots of all science observations made with the INT during the same period, categorised in the same way as **Figure 1**. **Figure 2** illustrates that the distribution of INT observations on the sky and with respect to instrument and phase of moon is similar to that of the WHT.

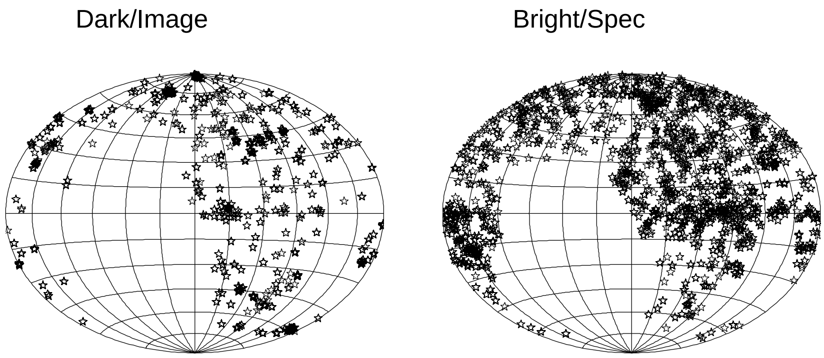

Figure 2. A plot of all INT science observations during the period 1997 August 1 to 1999 July 31, presented in Galactic coordinates. The projection is Aitoff and has the Galactic centre at the centre of the plot. Only imaging during *Dark* time and spectroscopy during *Bright* time are presented as a comparison with the WHT data.

2.2. Instrumentation used

From **Figures 1** and **2** it can be seen that many more spectroscopic observations are made with the INT and WHT than imaging. There are three main factors that may contribute to why this is the case:

1. Demand for imaging instrumentation is systematically lower than demand for spectroscopic instrumentation on these two telescopes.
2. Imaging programmes awarded time on these telescopes are less frequent than spectroscopic programmes.
3. Spectroscopic exposure times tend to be shorter than imaging exposure times.

Table 1 presents data showing number of operational hours used for science on each telescope, divided into moon phase periods, and presents the number of imaging and spectroscopic observations made during each period.

		Imaging			**Spectroscopy**		
		Dark	**Grey**	**Bright**	**Dark**	**Grey**	**Bright**
WHT	Observations	1499	926	695	5022	4214	11090
	Exposure Time	154.9	122.7	41.5	965.2	581.1	822.2
	Mean Exposure	372	477	215	692	496	267
	Nights	41	32	30	174	116	220
INT	Observations	3377	1803	1407	1503	2120	7393
	Exposure Time	425.9	191.3	131.8	237.1	289.7	776.7
	Mean Exposure	454	382	337	568	492	378

Table 1. Total number of observations, total exposure time in hours, the mean exposure time in seconds and the number of used nights for the INT and WHT. These data are categorised by instrument and phase of moon.

It is clear from the data presented in **Table 1** that the mean exposure times of the different classes of observations do not account for the high relative number of spectroscopic observations compared to imaging observations. In essence, this difference arises from the scheduling of observing programmes. The WHT has a ratio of spectroscopic

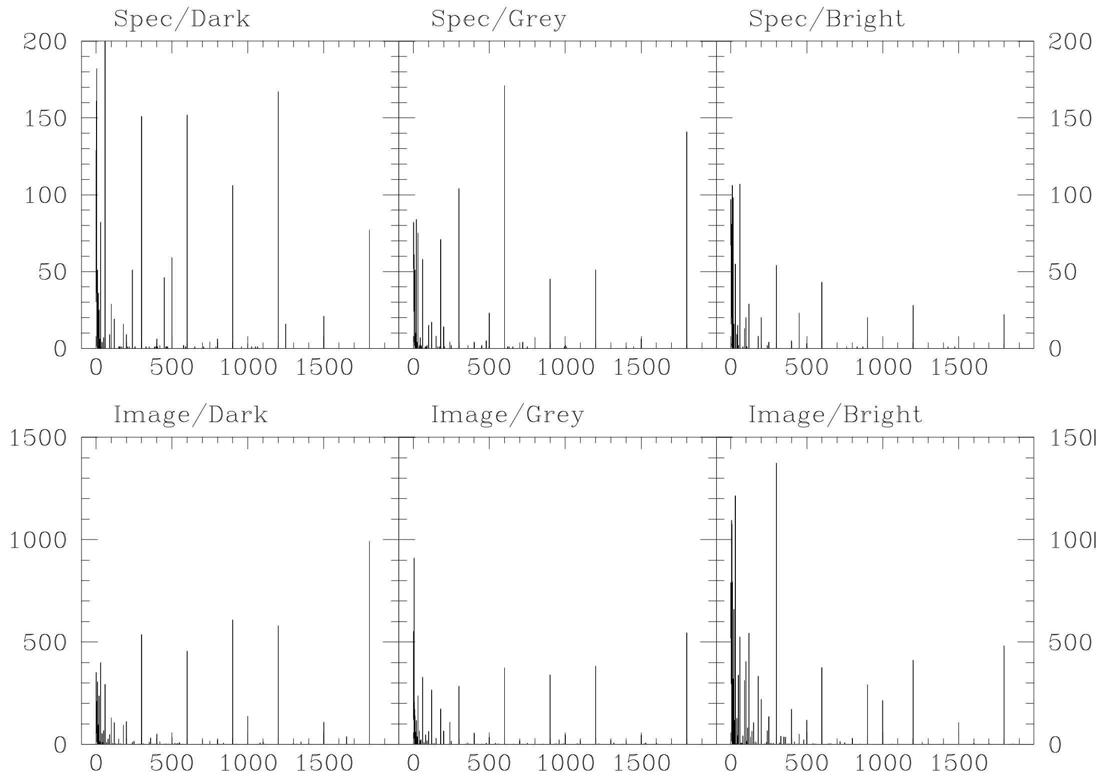

Figure 3. A plot of the exposure times in seconds of all WHT science observations during the period 1997 August 1 to 1999 July 31. The data are presented as frequency diagrams with a exposure time in seconds as abscissa. The bin size is intentionally small – 10 seconds.

versus imaging observations of greater than 7:1, whereas the INT has a ratio of 1:1.7. The WHT has two pure imaging instruments compared with seven spectroscopic instruments available to its user community, whereas the INT has one imaging instrument and one spectroscopic instrument. It is clear that the ratio of imaging to spectroscopic observations on the two telescopes reflects the available instrumentation on each telescope.

2.3. Mean exposure time

The mean exposure times for imaging and spectroscopic observations at each phase of the moon are presented in **Table 1**. It can be seen from the data in this table that mean exposure time shows a very similar trend on the WHT and the INT. In general, there is a tendency towards longer exposure times during *Dark* time. This trend reflects the effect of increasing sky brightness on the exposure times through its influence upon the nature of the science programme performed; *e.g.* for imaging observations, the goal of the effective use of *Dark* time is to image faint objects.

Figure 3 presents the exposure time data as histograms in the categories of the type of observation and the phase of moon. It is clear from **Figure 3** that exposure time is heavily quantised, with most longer exposures being quantised to integer multiples of 300 seconds. A second level of quantisation based upon integer multiples of 60 seconds is also apparent.

The evident quantisation of exposure time has clear implications for the efficiency of any observing programme. If exposure time is always rounded up to the next multiple of 300 seconds, then throughout one night many seconds of unnecessary exposure time have been used, when considered against using the optimum exposure time for a given signal/noise objective.

2.4. Operational implications

Several conclusions can be drawn from the results presented in this section:

1. The distribution of targets about the sky that a telescope will observe will vary, depending upon the basic type of observation; *i.e.* imaging or spectroscopy,
2. The frequency of a given type of observation performed by a telescope, *i.e.* imaging or spectroscopy, will reflect the range of instrumentation on the telescope.
3. For a general purpose telescope, imaging applications appear to be awarded systematically less telescope time than spectroscopy applications.
4. Observers generally do not optimise exposure time against signal/noise objectives that are derived from science goals. Instead, they tend to increment exposure times in multiples of seconds representing one or five minutes. The cumulative effect of this approach can be significant waste of exposure time throughout a night.

3. TELESCOPE EFFICIENCY

3.1. Definitions

In order to enable a quantitative discussion of telescope efficiency, it is first necessary to define the term with sufficient rigour that it can be used without ambiguity. In the remainder of this paper, the following definitions apply:

Reconfiguration:

We define *reconfiguration* to be a change of configuration of either the telescope (*e.g.* pointing the telescope at a new target) or the scientific instrumentation (*e.g.* a filter change). *Reconfiguration* is often called target acquisition or acquisition in the astronomical community.

Scheduling:

We define *scheduling* as being those processes that are required to determine the nature and sequencing of the next reconfiguration, possibly based upon the prevailing astronomical conditions and the quality of the existing data. Simplistically, reconfiguration means selecting the next target in the programme of observations.

Data Acquisition:

We define *data acquisition* as being all activities that lie between *reconfiguration* and *scheduling*. Two kinds of *data acquisition* exist: calibration (*e.g.* flat field exposures, wavelength calibration spectra), and astronomical. For both kinds of *data acquisition*, this process includes exposure, readout and data storage. We note that *data acquisition* is often used in the astronomical community to refer only to data readout from a detector and its storage on computer.

Observation:

We define an *observation* to be a single sequence of

1. Scheduling
2. Reconfiguration
3. Data acquisition

Availability:

The *availability* of a telescope system (including instrumentation) for a given period is the total time that it is capable of performing its defined science programme. Here, the *availability* of a telescope over a given time-scale may vary according to the nature of its science programme; *e.g.* infra-red versus visual band imaging or imaging versus spectroscopy. We also make the distinction between *ideal availability*, *i.e.* the maximum time at any given geographical location that a perfectly reliable telescope will be capable of performing its science programme, and *actual availability* which takes account of real environmental and reliability factors.

Ideal Detector:

We define an *ideal detector* as a device of unit detective quantum efficiency (DQE) over the operational wavelength range, with zero dark count and negligible readout noise.

Ideal Telescope:

We define an *ideal telescope* as a telescope system (including instrumentation) with an unobstructed aperture whose optics incur zero light loss and no optical aberrations in the image.

Efficiency:

The *efficiency* of a telescope system (including instrumentation) for a given period is the total time taken to reach a signal/noise goal during astronomical data acquisition divided by the telescope *availability*. This definition is intended to provide a numerical indication of how effectively an optical telescope is able to make use of its prime resource – the night sky.

3.2. Efficiency

The efficiency of a telescope, E, is given as

$$E = \frac{t_{\mathrm{DA}}}{A} \tag{4}$$

where

t_{DA}	=	Time consumed performing data acquisition.
A	=	Availability of the telescope system.

Here, t_{DA} and A are measured in the same units of time. The time consumed performing data acquisition is a function of both detector efficiency (DQE) and the optical efficiency of the telescope system (throughput), and can be separated into two components: time consumed acquiring calibration data and time consumed acquiring science data.

$$t_{\mathrm{DA}} = t_{\mathrm{DA}}(\mathrm{DQE}, \mathrm{Throughput}) \tag{5}$$

$$= t_{\mathrm{DA,Calib}} + t_{\mathrm{DA,Sci}} \tag{6}$$

The availability, A, of the telescope system may be enumerated in one of several ways, depending upon what design or evaluation criteria are being considered.

$$A_1 = t_{\mathrm{Dark}} - t_{\mathrm{Fail}} - t_{\mathrm{Met}} \tag{7}$$

$$A_2 = t_{\mathrm{Dark}} - t_{\mathrm{Met}} \tag{8}$$

$$A_3 = t_{\mathrm{Dark}} \tag{9}$$

where

A_1	=	Availability, accounting for the weather conditions of the telescope site and technical down-time.
A_2	=	Availability, accounting for only the weather conditions of the telescope site.
A_3	=	Availability, accounting for only the geographical siting of the telescope and its effect on the time the sky is dark enough to pursue the science programme.
t_{Dark}	=	The total time that the sky is dark enough to pursue the intended science programme.
t_{Fail}	=	The total time within t_{Dark} that the telescope is unavailable due to technical failures.
t_{Met}	=	The total time within t_{Dark} that the prevailing weather conditions prevent astronomical observations.

Here it is important to note that, for a given site, t_{Dark} will vary according to the requirements of the intended science programme. Furthermore, lower latitudes intrinsically provide more hours between the standard definitions of twilight, *e.g.* Nautical or Astronomical twilights, than higher latitudes.

We can define the total time utilised for astronomy, t_{Dark}, to be the sum of the time taken for scheduling, reconfiguration and data acquisition. This presents an alternative definition of availability; *e.g.*

$$A_1 = t_{\mathrm{Sched}} + t_{\mathrm{Recon}} + t_{\mathrm{DA}} - t_{\mathrm{Fail}} - t_{\mathrm{Met}} \tag{10}$$

where

t_{Sched} = The total time that the telescope consumes scheduling observations.
t_{Recon} = The total time that the telescope consumes reconfiguring the telescope system.

3.3. Signal/noise ratio for astronomical observations

There are two fundamental types of observation in astronomy – detection and measurement. In either case the criteria for success are defined in terms of the signal/noise ratio (SNR) of the observation (typically $SNR = 3$ for detection, $SNR > 10$ for measurement). Factors which affect SNR include telescope site (seeing and sky brightness), aperture, system throughput and exposure time. These equations are well known, and we do not lay claim to any originality for them. Nevertheless, it is useful to state them here for information.

The basic equation that defines the SNR obtained in a measurement with an array detector[2] is

$$SNR = \frac{S_{\mathrm{obj}} t}{\sqrt{S_{\mathrm{obj}} t + S_{\mathrm{sky}} t + D t + N R^2}} \tag{11}$$

where

S_{obj} = Object signal in detected electrons per spatial resolution element per second.
S_{sky} = Sky background in detected electrons per spatial resolution element per second
t = Exposure time in seconds
N = Number of detector pixels per spatial resolution element
R = RMS readout noise in electrons/pixel
D = Dark current in electrons/second

Equation (11) can be simplified in various limits. If $S_{\mathrm{sky}} >> NR^2$ and treating the dark current as an additional contribution to the sky background, we can write

$$SNR = \frac{S_{\mathrm{obj}} t}{\sqrt{S_{\mathrm{obj}} t + S_{\mathrm{sky}} t}} \tag{12}$$

i.e.

$$SNR = \sqrt{t} \times \frac{S_{\mathrm{obj}}}{\sqrt{S_{\mathrm{obj}} + S_{\mathrm{sky}}}}. \tag{13}$$

This is known as the sky limited case – the achieved signal/noise ratio for a constant telescope and instrument configuration increases as the square root of t. Such a case is often achieved with broadband imaging observations in the optical, and is nearly always true in the infra-red.

In contrast, optical spectroscopy is often read noise limited. In this case $(S_{\mathrm{sky}} + S_{\mathrm{object}}) << NR^2$. Therefore we can write

$$SNR = \frac{S_{\mathrm{obj}} t}{\sqrt{N} R}, \tag{14}$$

and the SNR increases in proportion to t.

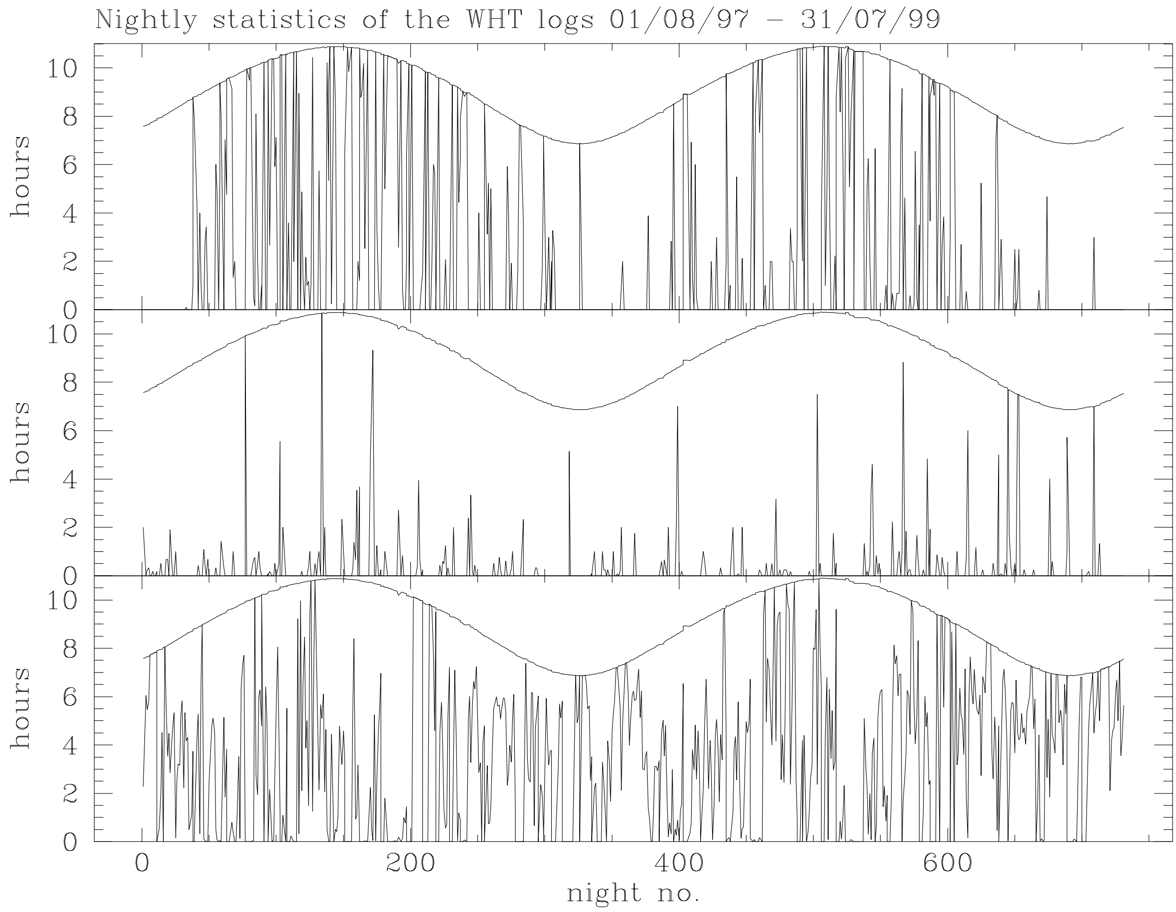

Figure 4. A plot presenting the number of hours used for astronomical exposures on the WHT during the period 1997 August 1 to 1999 July 31 (bottom), the number of hours lost due to technical failures (middle) and the number of hours lost due to poor weather conditions (top). The data are plotted with respect to night during the period. The top continuous line on each plot defines the total number of hours available for astronomical observations, as given in the ING logs.

4. FACTORS AFFECTING TELESCOPE EFFICIENCY

There are two principal ways in which telescope efficiency can be affected: by affecting telescope availability and by affecting the time used for astronomical data acquisition.

4.1. Availability

For a given latitude and science programme, the following factors will act to reduce the ideal availability:

1. Weather conditions
2. Telescope system reliability

Figure 4 presents plots of number of hours lost due to bad weather and technical failures on the WHT and also the number of hours used for astronomical observations. These plots also illustrate the number of hours potentially available for astronomical observations. It is clear from these plots that rarely do the number of hours used for astronomical observations equal the number of hours potentially available. **Table 2** presents data that confirms this numerically.

It can be seen from **Table 2** that losses of time due to bad weather and technical failures on the INT and WHT account for around 25%. However, the most remarkable observation is that only between 34.2% and 43.5% of time on these telescopes is used for gathering astronomical data. Of the rest, between 32.5% and 40.1%, can be attributed to scheduling, reconfiguration and image read-out. The remainder may be broken down as follows for the WHT[1]:

Telescope slews	2%
Object acquisition	4%
Issuing commands to the system	3%
Instrument mechanism movements	2%
CCD read-outs	6%
Image inspection and planning next observation	5%

These data account for 22% of the remaining 32.5% of telescope utilisation. Taking the contents of **Table 2** and the work of **Ref.** 1 allows the following values to be enumerated for the WHT:

Scheduling	8.5%
Reconfiguration	14.5%
Data acquisition	53.0%

where the 10.5% of telescope time still unaccounted for has been shared equally between the three categories.

The efficiency of the WHT can thus be calculated as

$$A_1 = 76.0\% \qquad \text{Efficiency} = 69.7\%$$
$$A_2 = 79.7\% \qquad \text{Efficiency} = 66.5\%$$
$$A_3 = 100.0\% \qquad \text{Efficiency} = 53.0\%$$

4.2. Data acquisition

Notwithstanding telescope availability, for a given telescope and science instrument the following factors will reduce the time available for astronomical data acquisition:

1. Scheduling time
2. Reconfiguration time (telescope and instrument)
3. Time taken performing calibration data acquisition

Telescope	Category	Percentage
WHT	Dark hours	100.0
	Science exposures	41.5
	Calibration exposures	2.0
	Lost to bad weather	20.3
	Lost to technical failures	3.7
	Remainder	32.5
INT	Dark hours	100.0
	Science exposures	32.5
	Calibration exposures	1.7
	Lost to bad weather	20.3
	Lost to technical failures	5.4
	Remainder	40.1

Table 2. Relative percentages of use of available astronomical time. The **Remainder** data are discussed further in the text.

4.2.1. Scheduling

It can be seen from the discussion in **Section 4.1** that scheduling time accounts for approximately 11% of the available astronomically useful telescope time. This time may be reduced by greater automation at the telescope, including automated scheduling and increased pipeline processing. The range of options for automating scheduling extends from simple queue observing, attended by the astronomer but planned prior to night-time operations, through flexible scheduling that accounts for current observing conditions to optimise science output from the telescope to fully robotic unattended operation. The trend towards more automated modes of scheduling within the astronomical community is indicative of the value of reclaiming this proportion of operational time.

4.2.2. Reconfiguration

It can be seen from the discussion in **Section 4.1** that reconfiguration time accounts for approximately 19% of the available astronomically useful telescope time. Reconfiguration may be subdivided into two major components: telescope reconfiguration and instrument reconfiguration. **Ref.** 1 breaks down the activities of what we term reconfiguration into the following activities: telescope slews, object acquisition (both telescope reconfiguration); instrument mechanism movements (instrument reconfiguration); issuing commands to the system (we equally divide between telescope and instrument reconfiguration). Hence, the proportion of telescope time spent on reconfiguration may be subdivided as follows:

Reconfiguration	19.1%	Telescope reconfiguration	13.0%
		Instrument reconfiguration	6.1%

The proportion of available astronomically useful telescope time may be reduced by the following measures:

1. Reduce telescope slew time.
2. Design instrumentation interfaces to minimise object acquisition time.
3. Design telescope and instrument user interfaces to minimise operator interaction times.
4. Design instrument mechanisms to operate concurrently with telescope slews.
5. Design instrument mechanisms to deploy systematically faster than telescope slew times.

If we take telescope slew times as an example, the WHT uses three axes during each typical slew: azimuth, altitude and the derotator for the focal station in use (*e.g.* Cassegrain). For the WHT, the maximum slew speed and the slew acceleration for each axis is $1.0° s^{-1}$ and $0.3° s^{-2}$ respectively. Using the order and target coordinates in the WHT logs, we determine that the WHT takes an average of $34s$ to slew from one target to the next. However, if the slew time of each axis is studied it is apparent that much of the mean slew time of the telescope is the result of the slew time for the Cassegrain derotator. These results are presented in **Table 3**.

	Maximum speed ($° s^{-1}$)			Acceleration ($° s^{-2}$)			Slew time (s)			
Case	**Alt.**	**Az.**	**Cass.**	**Alt.**	**Az.**	**Cass.**	**Alt.**	**Az.**	**Cass.**	**System**
WHT	1.0	1.0	1.0	0.3	0.3	0.3	19.5	5.1	26.2	33.6
WHT optimised	1.0	3.0	3.0	0.3	0.4	0.5	19.5	3.6	11.0	23.0
Fast optimised	2.0	3.0	3.0	0.3	0.4	0.3	11.9	7.7	12.4	17.0

Table 3. Mean slew times presented as functions of axis maximum slew speed and acceleration.

The data presented in **Table 3** suggest that there is an optimum combination of maximum slew speed and acceleration that results in a minimum slew time for the telescope system. If we assume that the azimuth maximum speed and acceleration are the most difficult to achieve technically and keep these constant, then the two remaining axes may be optimised to minimise the total slew time. Finally, if the maximum acceleration is doubled for the azimuth axis, the optimum parameters may again be determined for the remaining axes. The results of these models are also presented in **Table 3**. It can be seen from these results that the relationship between axes maximum slew speeds and accelerations and the mean telescope slew time is not a simple one. However, this example does indicate that careful design of subsystem performance can lead to significant system performance gains that have direct implications for telescope efficiency.

4.2.3. Calibration and science data acquisition

It can be seen from the discussion in **Section 4.1** that data acquisition time accounts for approximately 70% of the available astronomically useful telescope time. Data acquisition may be divided into science and calibration data acquisition. The proportion of telescope time spent on data acquisition may be subdivided as follows:

Data acquisition	69.7%	Science	66.5%
		Calibration	3.2%

There is a efficiency-based requirement to minimise calibration data acquisition during the available astronomically useful telescope time. Imaging calibration data may be acquired during twilight times or by using standard technical solution such as CCD underscan and overscan regions for bias determination. For adequate wavelength calibration to be performed only during twilight hours, spectroscopic instrumentation must be designed to render the effects of mechanical flexure negligible. This can be achieved either through minimising mechanical flexure at the design level, or by characterising instrumentation sufficiently to implement active compensation systems.

4.3. Exposure time

The following factors will affect the time taken to reach a signal/noise goal during astronomical data acquisition:

1. Telescope light grasp and throughput:
   - Telescope aperture
   - Telescope throughput
   - Atmospheric transparency
   - Wavelength resolution
2. Detector efficiency:
   - Detector quantum efficiency
   - Detector read noise
3. Sky brightness:
   - Moonlight (phase of moon)
   - Light pollution
4. Spatial resolution:
   - Atmospheric seeing
   - Local or dome seeing
   - Telescope tracking performance
   - Telescope aberrations

The effect of these factors upon the time taken to reach a signal/noise goal are discussed in the following sections.

4.4. Telescope light grasp and throughput

Referring to the signal/noise ratio discussion in **Section 3.3**, this section addresses the effects of telescope light grasp and throughput upon exposure times.

4.4.1. Telescope aperture

For a telescope of aperture diameter d, in both the read and sky limited cases for a given SNR

$$d^2 t = \text{constant}. \tag{15}$$

Unsurprisingly, a larger telescope implies shorter exposure times. More interestingly, this has important implications for proposals to build arrays many small telescopes. If the 'canonical' cube law for telescope cost versus aperture holds, then the weaker quadratic relationship between aperture and exposure time means that it better to build many small telescopes rather than one large telescope. However if the power of the cost law can be reduced to below quadratic then it becomes more cost effective to invest in a single large telescope.

4.4.2. Telescope throughput

S_{obj} and S_{sky}, the object signal and the sky background signal respectively, are proportional to the system throughput (E_t). This is given by

$$E_t = QE \times \prod_{x=1}^{x=X} T_x \tag{16}$$

where QE is the detector quantum efficiency, and T_x the transmission of component x.

In the sky limited case, SNR will be proportional to $\sqrt{E_t t}$. Similarly if we are read noise limited SNR will be proportional to $E_t t$. Therefore in either case for a given SNR

$$E_t t = \text{constant}. \tag{17}$$

For a reflecting telescope throughput is defined by three major quantities – the number of optical elements, the reflectivity of those elements, and the amount of obstruction of the beam due to the secondary mirror.

At 5500 Å a clean aluminium surface has a reflectivity of 0.88. For a two mirror aluminium telescope then $T_{\text{telescope}} \sim 0.77$, and a three mirror design (*e.g.* Nasmyth or folded Cassegrain) has $T_{\text{telescope}} \sim 0.68$. In contrast, untarnished silver has a reflectivity of 0.92 at 5500 Å, giving a three mirror $T_{\text{telescope}} \sim 0.78$ * (*i.e.* similar to the two mirror Aluminium design). Advances in coating technology mean that multiple layer coatings are now possible, especially for small mirrors, giving the potential of reflectivities of up to 0.97.

4.4.3. Atmospheric transparency

Atmospheric transparency will affect the SNR in the same way as telescope throughput, effectively reducing the anticipated signal from an object of known brightness.

4.4.4. Wavelength resolution

Wavelength resolution will affect SNR in the same fashion as telescope throughput. If $\delta\lambda$ is our wavelength resolution then

$$t\delta\lambda = \text{constant} \tag{18}$$

for a given SNR. This has implications for both choice of filter system in imaging (*e.g.* Sloan versus Johnson/Cousins filter systems) and resolution in spectroscopy.

4.5. Detector efficiency

4.5.1. Detector quantum efficiency

A modern CCD detector will have quantum efficiencies (QE) of typically $\sim$ 80% at around 6000 Å. However some detectors deliver much higher QE ($\sim$ 95%), and other detector technologies (*e.g.* superconducting tunnel junction detectors) are promise quantum efficiencies of $\sim$ 100%.

* at wavelengths < 3500 Å Silver is far inferior to Aluminium

4.6. Detector read noise

Here we are only concerned with the read-noise limited case. As is well known in this case, SNR is proportional to $1/R$, where R is the detector read noise. Hence for a given SNR

$$t/R = \text{constant}. \tag{19}$$

A typical astronomical CCD will tend to have an RMS read-noise of around 6 electrons If this can be reduced to 3 electrons, then exposure times can be halved.

4.7. Sky brightness

Both the effects of light pollution and moonlight are the same. Here, we need only consider the sky limited case. In addition, if we assume the object is faint with respect to the sky background (*i.e.* $S_{\text{obj}} << S_{\text{sky}}$) then we obtain

$$t/S_{\text{sky}} = \text{constant} \tag{20}$$

for a given SNR. Therefore a doubling of sky brightness with increase the exposure time similarly.

4.8. Spatial resolution

If it is assumed that we are interested in faint objects (*i.e.* of similar magnitude to the sky brightness), then we can make the approximation that the observations are sky-noise limited. Assuming a negligible dark current, the SNR is given by the simple formula

$$SNR = \sqrt{t} \times \frac{S_{\text{obj}}}{\sqrt{S_{\text{obj}} + a \times S_{\text{sky}}}} \tag{21}$$

where a is the area of spatial resolution element appropriate to the observation.

For point sources, spatial resolution is generally dictated by four principal factors:

- Site seeing (s_s)
- Facility (dome or local) seeing (s_f)
- Optical aberrations (s_a)
- Telescope tracking performance (s_t per second)

Assuming that the image spread (s_t) due to tracking errors is cumulative with time then we may combine these quadratically into a net seeing (FWHM=s) given by:

$$s = \sqrt{s_s^2 + s_f^2 + s_a^2 + (s_t t)^2} \tag{22}$$

Assuming the sky limited case, with $S_{\text{obj}} << S_{\text{sky}}$, and neglecting tracking errors then we can then approximate **Equation (21)** as

$$SNR \sim \sqrt{t/a} \times \frac{S_{\text{obj}}}{\sqrt{S_{\text{sky}}}}, \tag{23}$$

or, as $a \sim s^2$,

$$\frac{t}{s_s^2 + s_f^2 + s_a^2} = \text{constant}. \tag{24}$$

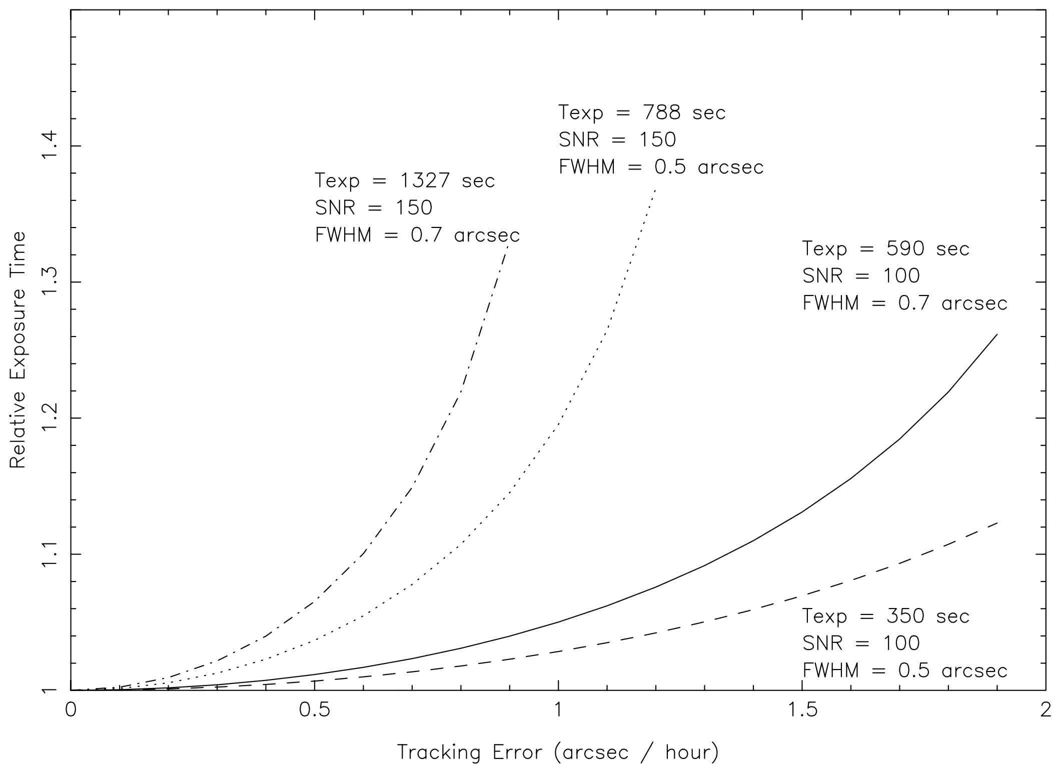

Figure 5. Effect of tracking error on exposure time for different seeing conditions and target SNR. Note how tracking accuracy is more critical in *poorer* seeing.

Exposure times are therefore governed by the second power of seeing, assuming small optical aberrations, and a small change in seeing will effect exposure times strongly. For instance an apparently small improvement in the net seeing from 0.7 arc seconds to 0.6 arc seconds will reduce exposure times by $\sim 25\%$. This implies that measures to reduce any facility seeing present will generally result in a considerable improvement in telescope efficiency. To take advantage of this improvement in efficiency implies the need for accurate measurement of the current seeing to allow exposure times to be optimally calculated. This can be achieved either by provision of a data reduction pipeline to extract the relevant information from the science data, or by use of independent seeing monitoring hardware (*e.g.* that based on the measurement of differential image motion over short time-scales). This also highlights the values of adaptive optics systems in increasing SNR, as they can potentially reduce the site seeing by factors of 2-3, decreasing exposure times for point sources by up to a factor of 10.

The effect of tracking accuracy on exposure times is a more complex problem. Substituting **Equation (22)** into **Equation (21)** and solving for t shows that

$$t = \frac{S_{\rm obj}^2 - \sqrt{S_{\rm obj}^4 - 4r^4 s_t^2 S_{\rm sky}((s_s^2 + s_f^2 + s_a^2)S_{\rm sky} + S_{\rm obj})}}{2r^2 s_t^2 S_{\rm sky}} \tag{25}$$

where $r = SNR$. To demonstrate the meaning of the equation we plot in **Figure 5** the relative increase in exposure time in the case of $S_{\rm obj} = 100 = 0.1 S_{sky}$ for signal/noise ratios of 100 and 150 and combined site and facility seeing values of 0.5 and 0.7 arc seconds. At first sight the results are counter-intuitive, as the effect of poor tracking is actually worse in poor seeing. In other words, it is more important to track accurately at a poorer site than a better one. This result can be understood however by considering the longer exposure times one obtains in poorer seeing leading to a greater image smear and therefore requiring a tighter tracking specification to obtain the same relative exposure degradation.

Item	Typical value	Optimal value	Efficiency gain
CCD QE	80%	95%	119%
Coating reflectivity	0.88	0.95	117%
Net seeing	0.7 arc sec	0.5 arc sec	196%
TOTAL			**273%**

Table 4. Net efficiency gain achievable with a 2-m telescope considering effects of detector QE, mirror coating and net (site + facility) seeing. Here we have assumed negligible optical aberrations in the telescope system.

We note here in passing that outside the sky-limited regime seeing is relatively unimportant. For example designing an optical spectrograph to make good use of poor seeing is a question of optical design to allow sufficient slit width – the disadvantage being the general requirement that wide slits imply large optics in order to achieve high wavelength resolution.

5. DISCUSSION

The essence of designing a telescope system for maximum efficiency lies in the **Equations (4)**, **(5)**, **(6)** and **(10)**. To summarise, the following steps need to be taken:

1. Maximise the amount of time on data acquisition with respect to scheduling and reconfiguration.
2. Minimise technical down time.
3. Minimise the amount of time required for calibration data acquisition during astronomically useful hours.
4. Maximise the detective quantum efficiency of the detector technology in use.
5. Maximise the throughput of the telescope and instrumentation in use.

Much of the above needs no great effort to comprehend. However, cost is also a key factor in telescope design. Using the equations presented above, in conjunction with cost estimates, will allow a simple cost-benefit analysis to be made of the various trade-offs involved in telescope and instrument design, including site selection. As a simple example, consider if for a particular telescope there was a choice of building a 2-m telescope at a "good site" (median seeing ~ 0.7 arc seconds) or a 3-m telescope at a poorer site (median seeing 1.2 arc seconds) for the same cost. The relative efficiency of the 3-m over the 2-m in terms of aperture is 225%. However the relative efficiency of the 2-m over the 3-m in terms of seeing is 293%, leaving the 2-m with a net efficiency of 130% over the 3-m. Alternatively given the same choice of aperture and seeing but brighter 0.7 mag/square arc second in sky brightness, then the 3-m would be the most efficient option (efficiency $\sim 112\%$ with respect to the 2-m).

It is interesting to compare the effect of "typical" values for the parameters discussed in the preceding sections versus the optimal values that may be obtained in order to derive the total efficiency gain that may be achieved by attention to a number of parameters. In **Table 4** we therefore present the typical and optimal values of some of those parameters for a 2-m two mirror telescope equipped with an imaging CCD Camera. Considering only the effects of quantum efficiency, mirror coating and site seeing, we see that we may obtain an efficiency gain of $\sim 270\%$, equivalent to an increase in aperture from 2-m to 3.3-m.

6. CONCLUDING REMARKS

We have developed a set of definitions and a formulation by which the efficiency of an astronomical telescope can be evaluated and therefore optimised as part of a design process. Operational data from the Isaac Newton Group have provided insight into actual operational behaviour of several of the derived factors affecting telescope efficiency. We have explored the implications of this formulation when used to evaluate the effectiveness of telescope designs.

Clearly, the equations presented need to be used alongside telescope cost calculations in order to optimise the design of telescopes for cost-effectiveness within their application areas. Also, as in any optimisation process, it is important to note risk of over-optimisation. The future use of any telescope means that any optimised telescope design can be used in ways not foreseen at procurement. Over-optimisation may limit the future usefulness of a telescope.

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